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CALCULATION OF THE THERMAL STABILITY OF A LIQUID-METAL CONTACT IN AN EMERGENCY CONDITION

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The problem of finding the maximum temperature of an infinite plate with adiabatic boundaries is solved by the method of image sources. The solution obtained is used to calculate the heating of a liquid-metal contact in an emergency condition and to find a suitable solid-metal wall thickness.

Electrical contacts between liquid and solid metals are finding increasingly wide application [1–4]. In an emergency short-circuit condition, the current passing through the solid metal–liquid metal contact may be several times or even several tens of times greater than the rated value.

At the end of the short circuit, when the circuit becomes disconnected, the temperature in the contact zone may exceed the permissible value for the materials used in liquid-metal contacts.

The thermal stability of a contact is understood as its ability to withstand a current of constant strength for a finite time in such a way that the temperature at the end of that time does not exceed the permissible value.

It is evident that the maximum temperature will occur in the plane of contact of the liquid and solid metals, since a heat source of specific power N_0 is located in precisely that plane. Here

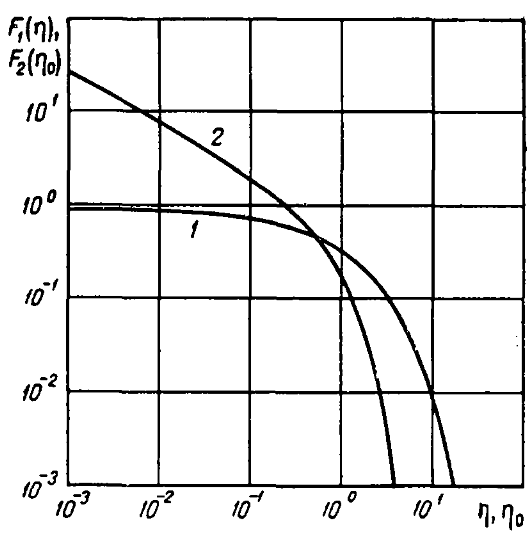
$$N_0 = \delta^2 \sigma. \tag{1}$$

In the first approximation, the temperature of the contact may be determined as the temperature at the boundary of a semi-infinite body, if a uniformly distributed specific heat source N_0 constant in time is introduced at that boundary. Then the boundary condition may be written as

$$-\lambda \frac{\partial T}{\partial x}\bigg|_{x=0} = N_0 . \tag{2}$$

In this formulation of the problem it is assumed that the contact area is the same as the geometrical area of contact of the solid with the liquid, and that the current is uniformly distributed over the contact surface. It is additionally assumed that the liquid metal layer is so thin that all the heat liberated at the contact is absorbed by the solid. Fringe effects due to the finite size of the contact in the plane of contact of liquid and solid are not considered.

The solution of the heat conduction equation with the above assumptions and boundary condition (2) is known [5] and may be expressed as



Graphs of functions $F_1(\eta)$ – 1 and $F_2(\eta_0)$ – 2.

$$T_1(x,\ t) - T(0,\ 0) = F_1(\eta)\, 2N_0 \sqrt{t} \,/ \sqrt{\pi c \rho \lambda}, \tag{3}$$

$$F_1(\eta) = \exp\left(-\frac{1}{4}\eta\right) - \frac{1}{2}\sqrt{\pi\eta}\, \mathrm{erfc}\left(\frac{1}{2}\sqrt{\eta}\right), \tag{4}$$

$$\eta = x^2/at. \tag{5}$$

The function $F_1(\eta)$ is shown graphically in the figure.

When the thickness of the solid part of the contact in a direction perpendicular to the plane of contact of liquid and solid is small, Eq. (3) gives a certain error. A more accurate solution may be obtained by examining the problem of a plane heat source of specific power $2N_0$ acting continuously for time t and located in the plane of symmetry of an infinite plate of thickness 2d.

In actual conditions, at the boundaries $x = \pm d$ of the plate we get heat transfer to the surrounding medium, which may be air, water, or oil, depending on the type of construction. However, since the time of operation of the contact in an emergency condition is of the order of some seconds or fractions of a second, we may, with sufficient accuracy, consider the above boundaries to be adiabatic. In that case the boundary conditions at $x = \pm d$ may be written as

$$\mp \lambda \frac{\partial T}{\partial x}\bigg|_{x=\pm d} = 0. \tag{6}$$

And in the plane of symmetry of the plate

$$-\lambda \frac{\partial T}{\partial x}\bigg|_{x=0} = N_0 . \tag{7}$$

The solution of this problem may be obtained by the method of image sources. Applying this method successively to the boundaries $x = \pm d$, we obtain an infinite medium with uniformly distributed plane sources of specific power $2N_0$ at distances 2d apart. The temperature distribution due to one such source is described by (3). Summing the effect of all these sources at $x = 0$, where the temperature has a maximum, we obtain

$$T_2(0,\ t) = T(0,\ 0) = \frac{2N_0 \sqrt{t}}{\sqrt{\pi c \rho \lambda}} \left[1 + 2\sum_{n=1}^{\infty} F_1(\eta_n)\right], \tag{8}$$

where

$$\eta_n = (2dn)^2/at = (2n)^2 \eta_0 , \tag{9}$$

$$\eta_0 = d^2/at. \tag{10}$$

Introducing the notation

$$F_2(\eta_0) = 2\sum_{n=1}^{\infty} F_1(4n^2 \eta_0), \tag{11}$$

we obtain

$$T_2(0,\ t) - T(0,\ 0) = \frac{2N_0 \sqrt{t}}{\sqrt{\pi c \rho \lambda}} [1 + F_2(\eta_0)]. \tag{12}$$

A graph of function $F_2(\eta_0)$ is also shown in the figure.

When $\eta_0 > 2$, function $F_2(\eta_0)$ takes a value not exceeding 0.05. This means that to an accuracy of 5% the influence of the adiabatic boundaries at $x = \pm d$ on the temperature in the plane $x = 0$ may be neglected.

We note that as regards thermal stability of the contact, the wall with $\eta_0 = 2$ is more suitable, since an increase in its thickness does not entail an appreciable lowering of the maximum temperature in the plane of contact, i.e., at $x = 0$. The table gives suitable values of the wall thickness d_{01} for various materials and a heat flux acting for 1 sec. (If the time during which the heat flux acts is t, $d_{0t} = d_{01}\sqrt{t}$).

Thermophysical Constants of Materials and Suitable Wall Thicknesses

Material	$c \cdot 10^{-3}$, J/kg · deg	$\rho \cdot 10^{-3}$, kg/m^3	$\lambda \cdot 10^{-2}$ j/m · ·sec ·deg	$a \cdot 10^4$, m^2/sec	$d_{01} \cdot 10^2$, m
Copper	0.389	8.9	3.850	1.100	1.483
Brass	0.385	8.6	1.090	0.329	0.811
Bronze	0.375	8.2	1.10	0.375	0.845
Beryllium bronze	0.375	8.4	0.503	0.160	0.566
Carbon steel	0.448	7.8	0.461	0.132	0.514
Stainless steel	0.449	7.9	0.170	0.044	0.296

It follows from the above that when $\eta_0 \geq 2$ the contact temperature in an emergency condition may be calculated assuming that the medium into which the heat flux is released is infinite in extent, if the required accuracy of the calculation does not exceed 5%. When greater accuracy is required, we may use (10), in which the series (11) converges rapidly for $\eta_0 > 0.5$.

If, because of design considerations, the ratio of wall thickness d and time of operation in the emergency condition are such that $\eta_0 < 0.5$, it is more convenient to use the solution given in [5], which in our notation may be written as:

$$T_2(0, t) - T(0, 0) = \frac{2N_0\sqrt{t}}{\sqrt{\pi c \rho \lambda}} \frac{\sqrt{\pi}}{2} \frac{1}{\eta_0} \times$$

$$\times \left\{ 1 + \eta_0 \left(\frac{1}{3} - 2 \sum_{n=1}^{\infty} \frac{\exp(-n^2 \pi^2 \eta_0^{-1})}{n^2 \pi^2} \right) \right\}. \quad (13)$$

Values of function $F_2(\eta_0)$ for $\eta_0 < 2$ have been calculated from this formula and are shown in the figure. It can be seen that they agree with the values obtained from (11).

NOTATION

x—distance from the plane containing the source to the point at which the temperature is sought; t—time; λ—thermal conductivity; c—specific heat; ρ—density; δ—current density; σ—resistance per unit area of contact; a—thermal diffusivity; T(0, 0)—initial temperature; T(x, t)—temperature at a point in the investigated medium; η—dimensionless parameter, inverse Fourier number

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